

## A CBS-FINITE ELEMENT METHOD FOR COMPLEX FLOWS OVER FORESTED RELIEFS

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**Abstract.** *In this work a numerical methodology combining a finite element approach and CBS (Characteristic Based Splitting) method is used to simulate canopy flows non-homogeneous forests. The particular interest is to simulate the atmospheric fluid flow over complex relieves, or in a proximity of discontinuity of vegetative layer distribution (clearings or rivers). A turbulence model is used to describe the flow within the forest canopy. This model uses a first order closure approach, with a computation of the eddy viscosity by an algebraic model. The vegetative drag force effects are taken into account by means of a quadratic term. The computation of velocity statistics is performed using anisotropy estimative of the distribution of kinetic energy of turbulence. The implementation of the CBS algorithm for the turbulence model was performed in a 2D code that use triangular elements with an equal order of velocity and pressure interpolation. Results for homogeneous forest, forest edges and clearings are presented and compared in situ measurements.*

**Keywords** canopy flows, CBS finite element, turbulent boundary layer.

### 1. Introduction

The description of the atmospheric turbulent air flow within and above vegetative canopies has become an important research interest, which has been associated to comprehension and quantification of physicochemical and ecological processes in forests and plantations. The modeling of this kind of turbulent flow is inserted in a perspective of the advanced closing of statistical turbulent moments in averaged fluid mechanics governing equations, and the use of the this models to simulate real and complex situation have to be associated to the properly numerical implementations.

On the last decade a great interest for the numerical solutions of inhomogeneous canopy flow problems has arisen (Lee, 2000), involving, for instance, situations like flows through forest cut blocks (Wilson and Flesh, 1999) or over complex forested relieves (*e. g* Kobayashi et al., 1994, Ross et al., 2004 or Ross and Vosper, 2004). For this kind of problem, advanced turbulence models considering the vegetative canopy layers have been proposed, and its numerical implementation, using algorithms that can treat complex geometries, have to be employed. A real complex flow through and over forested relieves occurs in a domain with a bottom surface issued from the topography of geographic regional terrain database. Sometimes, great gradients of elevation are encountered, and over this relief the vegetation is often distributed non-uniformly, due to different structures of the vegetative layer and to the existence of clearings.

The use of a detailed description of vegetative elements individually is impossible to be considered in this scale of analysis, and the modeling of the vegetative layer has to be represented in a sense of a local spatial averaging procedure. The actual approach for simulating canopy flows considers the averaging the flow variables within the forest layer, using a physical control volume of an order of some meters in the vertical direction and hundred of meters in the horizontal direction. On this scale, one can evaluate some features of the flow related to the local variations. It can allow the comprehension and quantification of heat and mass exchanges between the vegetation and the atmosphere.

The turbulence models issued from this averaging approach have some distinct characteristics from the conventional models for free-flows, and the numerical methodologies have to be adapted to this new condition. Considering this averaging process for the fluid flow modeling, the closing of the governing equations can either use second or first-order models.

The present paper proposes a simple modeling approach based on first-order with algebraic eddy viscosity. An alternative approach to compute the length scale non-homogeneous canopy situations is explored.

In a context of first-order turbulence closing models (K-1 or K- $\epsilon$ ), it has already been shown that these simple models can accurately describe both the mean velocity and the kinetic energy profiles in homogeneous forest boundary layers (Katul et al., 2004; Pinnard and Wilson, 2001).

In some non-homogeneous vegetation distribution, like in forest cutblocks for instance (Wilson and Flesh, 1999), the numerical results show good agreement with the field data, taken into account some corrections of the internal parameters of the model. The low computational cost of the first-order models, in particular for 2D or 3D problems, is the great motivation of the development of this class of models.

The aim of the present paper is to present an alternative zero-equation model for canopy flows. A canopy flow model was implemented in a finite element code using the *Characteristic-Based Splitting* approach, with P1-P1 triangular elements. The present finite element model also uses the same order of interpolation for the pressure and velocity fields, considering the self-stabilization behavior of the CBS algorithm by means of the time step control.

This paper is organized as follows: In the section 2 the modeling approach is described and the specifically closing assumptions are discussed. In the section 3 the numerical implementation of the turbulence model is presented and the stability consideration is explored. At a last part, some illustrative results are presented concerning the flow over different forest configurations, where experimental data is compared to the present numerical results.

## 2. Turbulence modeling

### 2.1 Basic Equations

Let us consider a multidimensional air flow within and above vegetative canopies, defined for generality in a 3D domain. The modeling of canopy turbulent flows considers a *rationale* based in a double averaging process: In a first step all flow quantities are averaged on time using a characteristic time interval, like for free turbulent flows (this averaging operator being denoted by a over line bar symbol  $\overline{(\cdot)}$ ). In the second step a spatial averaging is performed within a reference volume (this operator being denoted by a bracket symbol  $\langle(\cdot)\rangle$ ). Discussions of the use and necessity of this formalism are developed in Raupach and Shaw (1982) or Finningan (2000), for instance.

Applying this double averaging approach to the instantaneous mass and momentum conservation equations for the air flow, the set of governing equations can be obtained. The assumptions of incompressibility of the air flow under neutral buoyancy conditions and no-waving behavior of canopy elements were considered. The continuity and the Navier-Stokes equations for the canopy flows are thus given by:

$$\nabla \cdot \langle \overline{\mathbf{u}} \rangle = 0 \quad (1)$$

$$\frac{\partial \langle \overline{\mathbf{u}} \rangle}{\partial t} + \langle \nabla \langle \overline{\mathbf{u}} \rangle \langle \overline{\mathbf{u}} \rangle \rangle = -\frac{1}{\rho} \nabla \langle \overline{p} \rangle + \nabla \cdot \left( 2\nu \mathbf{S}(\langle \overline{\mathbf{u}} \rangle) - \langle \overline{\mathbf{u}' \otimes \mathbf{u}'} \rangle - \langle \mathbf{u}'' \otimes \mathbf{u}'' \rangle \right) + \mathbf{f} \quad (2)$$

In these equations  $\langle \overline{\mathbf{u}} \rangle$  and  $\langle \overline{p} \rangle$  are the averaged velocity and pressure fields,  $\rho$  and  $\nu$  denote the density and kinematical viscosity of the air and  $\mathbf{S}(\langle \overline{\mathbf{u}} \rangle)$  is the mean rate of strain tensor given by:

$$\mathbf{S}(\langle \overline{\mathbf{u}} \rangle) = \frac{1}{2} \left( \nabla \langle \overline{\mathbf{u}} \rangle + \nabla^T \langle \overline{\mathbf{u}} \rangle \right) \quad (3)$$

The extra terms of Eq. (2), present due to the use of double averaging operations on the momentum conservation equation, are the volume averaged Reynolds stress  $\langle \overline{\mathbf{u}' \otimes \mathbf{u}'} \rangle$  and the dispersive flux  $\langle \mathbf{u}'' \otimes \mathbf{u}'' \rangle$ . These terms denote the statistical correlation of fluctuating velocity parts associated to the time average,  $\mathbf{u}'$  and to the space,  $\overline{\mathbf{u}''}$ , respectively. In the present work the tensor term related to the dispersive flux is neglected, considering the density of the canopy elements for some kinds of forest structures it can be taken into account.

The last term in the Eq. (2),  $\mathbf{f}$ , accounts for the mechanical interaction between the air flow and the vegetation elements by viscous and form drag forces. It can be modeled by:

$$\mathbf{f} = -C_D A(x_3) \langle \overline{\mathbf{u}} \rangle \langle \overline{\mathbf{u}} \rangle \quad (4)$$

Where  $C_D$  and  $A(x_3)$  denote respectively the drag coefficient and plant area density, with  $x_3$  coordinated aligned to the vertical direction.

### 2.2. Closing parameterization

In the framework of first-order turbulence closing, the Reynolds stress tensor is often modeled by the Boussinesq eddy-viscosity assumption, namely

$$\langle \overline{\mathbf{u}' \otimes \mathbf{u}'} \rangle = \frac{2}{3} K \mathbf{I} - 2\nu_T \mathbf{S}(\langle \overline{\mathbf{u}} \rangle) \quad (5)$$

where  $K$  represents the kinetic energy of turbulence (which will be combined to compose the apparent pressure) and  $\nu_T$  is the eddy viscosity. In the present paper a zero-equation model is employed as:

$$\nu_T = \ell_m^2 \left[ \mathcal{S}(\langle \bar{\mathbf{u}} \rangle) : \mathcal{S}(\langle \bar{\mathbf{u}} \rangle) \right]^{1/2} \quad (6)$$

in which  $\ell_m$  represents the mixing length of the turbulent process. For homogeneous forest it can modeled simply as:

$$\ell_m = \begin{cases} \ell_h & ; \text{if } z \leq h^* \\ \kappa(z-h^*), & \text{if } z > h^* \end{cases} \quad (7)$$

In the above equation  $h^*$  is the vertical position where the leaf area density is maximum ( $A_{max}$ ) and  $z$  is the vertical distanced of the closest ground surface. The length scale for the canopy region  $\ell_h$ , is given as

$$\ell_h = \frac{\kappa h^*}{1.5 - 2.5 A_{max}} \quad (8)$$

where  $\kappa=0.44$  represents the Karmann constant.

### 2.3 Boundary conditions

Four types of boundary condition are considered for the atmospheric boundary layer flow simulations. On the ground non-slip boundary conditions is imposed for the velocity flow. On the free flow surface, located three times the forest height, the free velocity is imposed. For the problems related to a 2D evolution of the boundary layer (forest edge, for instance) the inflow condition is composed by a vertical profile of the velocity field, based in a power-law atmospheric boundary variation. For the outflow boundary condition homogeneous Neumann conditions for the velocity field and a reference value for the pressure are imposed.

### 3. Numerical methodology

The governing equations presented on the last section are solved by employing a finite element method with an equal order interpolation for pressure and velocity fields (linear triangle elements have been selected). A CBS algorithm is implemented taking into account the regions with and without forest canopies, always for 2D domains. In this methodology the continuity and momentum equations are solved using a splitting strategy, considering an incremental time integration algorithm with multiple steps. This approach can assure the stability for pressure and velocity, as well as for high local Reynolds number, only by controlling the size of the time step.

The aim of the CBS algorithm involves two major ideas (e.g. Zienkiewicz and Taylor, 2000, Codina et al., 1998): First the momentum equation is re-written along a characteristic path, in order to reduce the spurious effects due to Galerkin discretization for high Reynolds number. This gives rise to an additional stabilized term in the formulation on streamline direction, equivalent to the streamline-diffusion term. The second feature of this algorithm is to decouple the pressure and velocity fields by means of a fractional step algorithm, like in classical splitting-projection schemes (Chorin, 1968 and Temann, 1969). It is shown that this last approach allows a stabilization term for the pressure and velocity discretization spaces (Codina, 2001). Those two ingredients of the method permit a stable scheme for convective-advection treatment and for pressure-velocity discretization. The stabilization parameter now is the time step. It can be shown that this scheme has equivalent stabilized properties of other methodologies (Codina and Zienkiewicz, 2002).

Given the set of variables known in a previous time step  $t$ ,  $\{\mathbf{u}^n, p^n\}$ . The solution  $\{\mathbf{u}^{n+1}, p^{n+1}\}$  of the conservation equations in a time step  $t + \Delta t$ , is obtained by the following steps:

#### Step 1: Solving Momentum Equation

$$\Delta \mathbf{u}^* = \mathbf{u}^* - \mathbf{u}^n = \Delta t \left[ -\mathbf{u}^n \nabla \cdot \mathbf{u}^n + \nabla \cdot ((\nu + \nu_T) \nabla \mathbf{u}^n) + \mathbf{f}(\mathbf{u}^n) \right] + \frac{\Delta t^2}{2} (\mathbf{u}^n \cdot \nabla) \nabla \cdot \left[ (\mathbf{u}^n \otimes \mathbf{u}^n) - \mathbf{f}(\mathbf{u}^n) \right]$$

#### Step 2: Solving Pressure Field

$$\nabla^2 p^{n+1} = -\Delta t \left[ \nabla \cdot \mathbf{u}^n + \nabla \cdot \Delta \mathbf{u}^* \right]$$

**Step 3: Velocity Correction – Divergence-Free Projection**

$$\mathbf{u}^{n+1} = \mathbf{u}^n - \Delta t \nabla p^{n+1}$$

The term  $\mathbf{f}(\mathbf{u})$  is computed by using Eq. (4).

Using the Galerkin method for spatial discretization of the equations on the steps 1-3, coupled to the classical finite element base functions, a matrix form of the algorithm can be written as set of three symmetrical linear systems for each time step.

$$\begin{cases} \mathbf{M} \cdot \Delta \mathbf{u}^* = \mathbf{f}_u^* \\ \mathbf{H} \cdot \mathbf{p} = \mathbf{f}_p \\ \mathbf{M} \cdot \Delta \mathbf{u} = \mathbf{f}_u \end{cases}$$

In those equations  $\mathbf{M}$  and  $\mathbf{H}$  are the mass and discrete Laplacian matrices given by:

$$\mathbf{M}_{ij} = \int_{\Omega} N_i N_j d\Omega \quad ; \quad \mathbf{H}_{ij} = \int_{\Omega} \nabla N_i \cdot \nabla N_j d\Omega$$

**Remarks:**

- The linear systems of the steps 1 and 3 involve the mass matrix. In order to enhance the convergence rate of the computations, this matrix is lumped in a diagonal form. This is performed only once, at the beginning of the iterative computation.

- The linear system for the pressure correction problem (step 2) is solved by employing the Conjugated Gradient Method, preconditioned by partial Cholesky factorization. This matrix is stored by means of a sparse Morse strategy, and the preconditioning is also performed only once, when this matrix is firstly computed.

In the present paper the time step is controlled by the following expression:

$$\Delta t \leq \frac{\Delta t_c \Delta t_d}{\Delta t_c + \Delta t_d} \tag{9}$$

where

$$\Delta t_c = \left( 2 \|\mathbf{u} \cdot \nabla N_i\|_{\max} \right)^{-1} \quad ; \quad \Delta t_d = \left( 2 \nu^e \|\nabla N_i \cdot \nabla N_j\|_{\max} \right)^{-1} \tag{10}$$

For those two expressions ones considers the characteristic time for the diffusion and convection counterpart of the discrete problem, at each element. The viscosity in the element,  $\nu^e$ , must take into account the molecular and turbulent parts.

It can be verified that the critical time step proposed by Eq. (9) is compatible with the following relation:

$$\Delta t \leq \left( c_1 \frac{\nu^e}{h^2} + c_2 \frac{|\mathbf{u}^n|}{h} \right)^{-1} \tag{11}$$

where  $c_1$  and  $c_2$  are constants and  $h$  denotes the characteristic length of the element. The above described procedure can assure the stability of the scheme, following the analysis of Codina and Zienkiewicz (2002). The present choice of the time step value stabilize both the convective and pressure-velocity problems.

**4. Results and discussions**

The first test case concerns the 1-D developed boundary layer over and within a homogeneous canopy. This problem is used to check the quality of the simulations using the simply first order model for different density canopy distributions. It is considered that the forest canopies have the height  $H$ . The computational domain for the canopy problem is considered  $3H$  height, where a free velocity is imposed. A non slip boundary condition is used on the ground. All general characteristics of those forests are resumed in Tab. 1, and the leaf area density of forest sites are presented in the Fig. 1.

Table 1: General forests characteristics

Forest type	Drag Coefficient ( $C_D$ )	Height (H)	Leaf Area Index (LAI)	Reference
Amazon Rain Forest	0.25	38 m	4.9	Kru et al (2000)
Pine Forest	0.21	14 m	2.9	Katul et al (2004)
Oak Forest	0.1	24 m	2.9	Poggi et al (2004)
Stika Spruce	0.2	7 m	2.15	Irvine et al (1997)

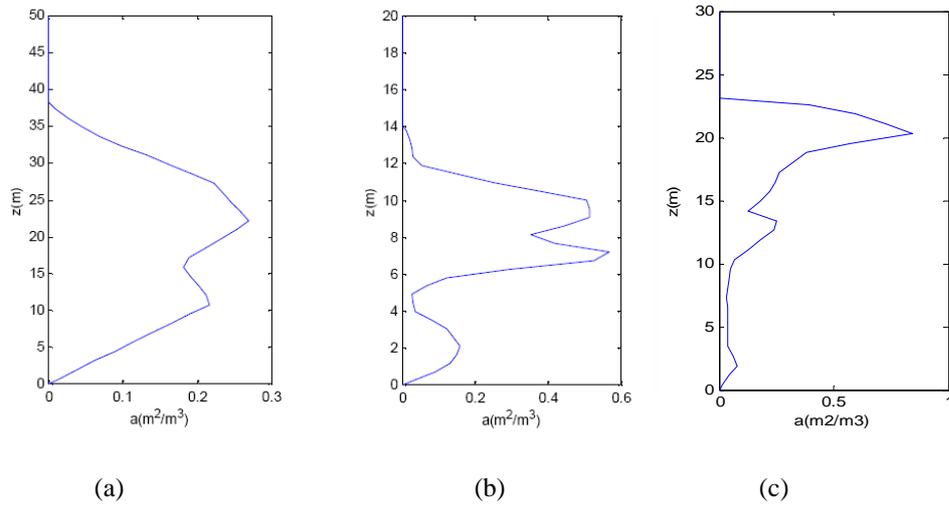


Figure 1. Leaf Area Density: (a) Amazon Rainforest (b) Pine Forest (c) Oak Forest.

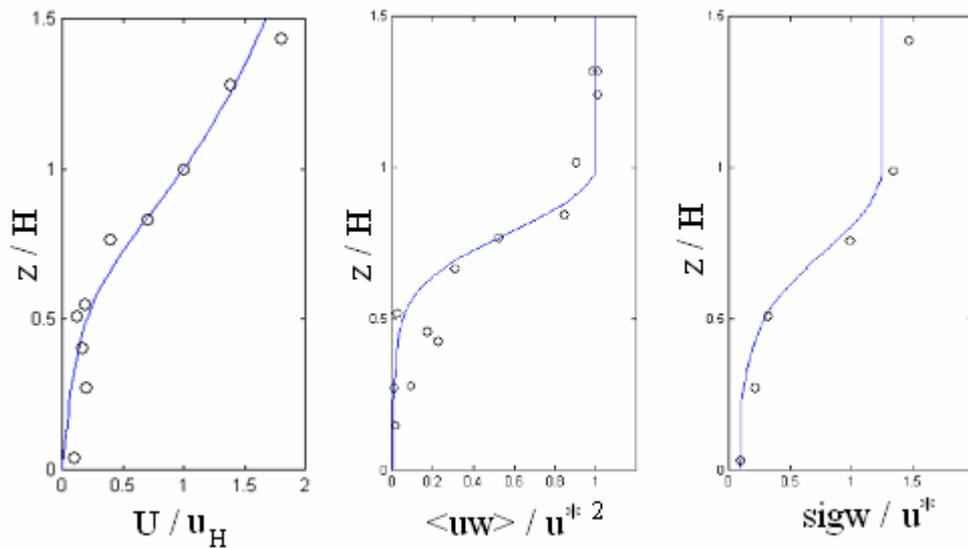


Figure 2. Developed boundary layer for Amazon rainforest : Velocity and turbulence profiles.

The results for developed flow – after the boundary layer development, are presented in Figs. 2 and 3, in which the numerical results are compared to the experimental measurements for Amazon and pine forests. The mean horizontal velocity profiles for both simulations could reproduce well the experimental data. The turbulence statistics given by the

Reynolds Shear stress ( $\overline{u'w'}$ ) and the vertical velocity variance ( $\sigma_w$ ) are obtained by using the following simply parameterizations:

$$-\langle \overline{u'w'} \rangle = \nu_T \left( \frac{\partial \langle \overline{u} \rangle}{\partial z} + \frac{\partial \langle \overline{w} \rangle}{\partial x} \right) \tag{12}$$

$$\sigma_w = \sqrt{\frac{2}{3}} k ; k = 2.357 \frac{\nu_T^2}{\ell_m^2} \tag{13}$$

All the displayed results are presented by considering dimensionless values, using the velocity scales ( $U^*$  or  $U_H$ ) proposed by the experimental works. The results obtained in the simulations show good agreement with the *in situ* measurements, considering the simplicity of the first order model used in the present paper. In Fig. 4 a complete comparison between the *in situ* experiments and numerical simulations is presented for the mean velocity field as well as for the turbulent shear stress. These plots have shown the relative quality of the proposed model. The model limitation for the high velocity values is observed in the spread of the data in the upper side of the graphics, mainly for the simulations of the shear stress. The main dispersion is encountered in the simulations of lower density canopies (oak) over the canopy.

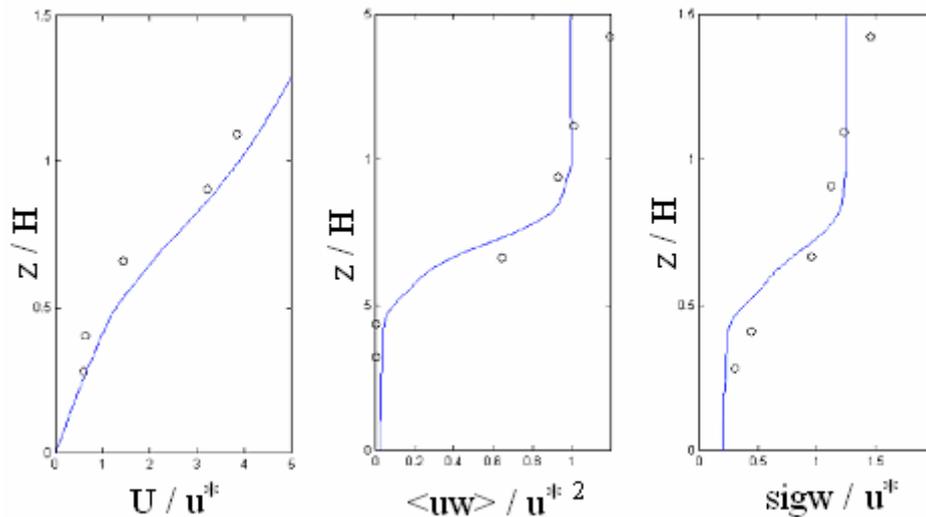


Figure 3. Developed boundary layer for Pine Forest: Velocity and turbulence profiles. (symbols are experimental measurements. Lines are simulations)

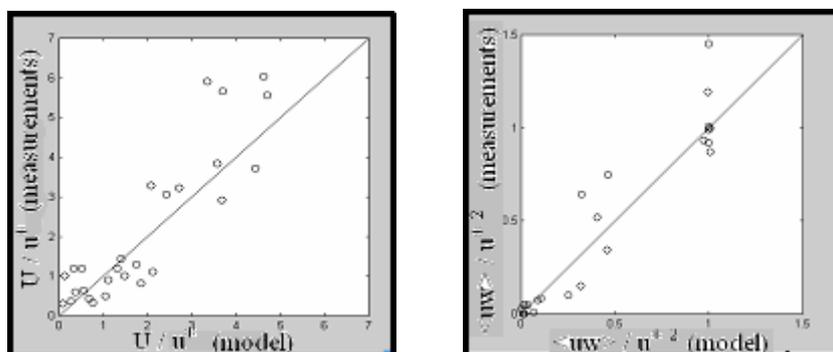


Figure 4. Comparisons for four dense forests (mean velocity and shear stress).

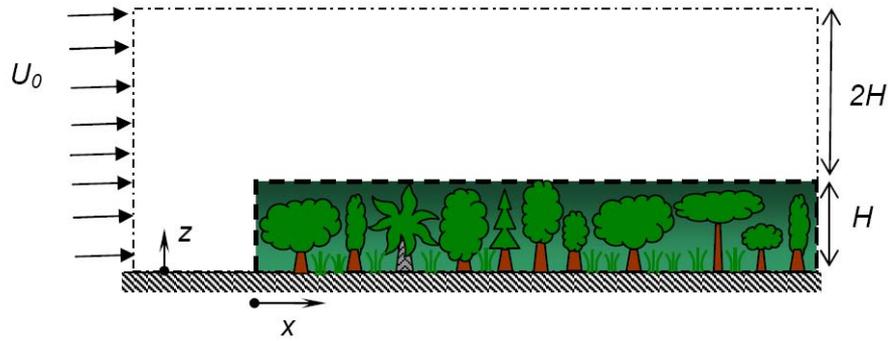


Figure 5. Forest edge test case.

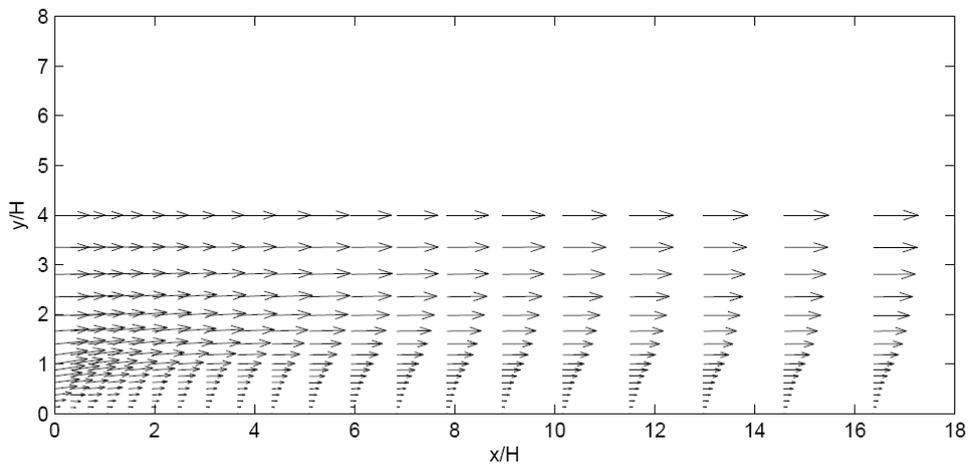


Figure 6. Boundary layer development from the forest edge.

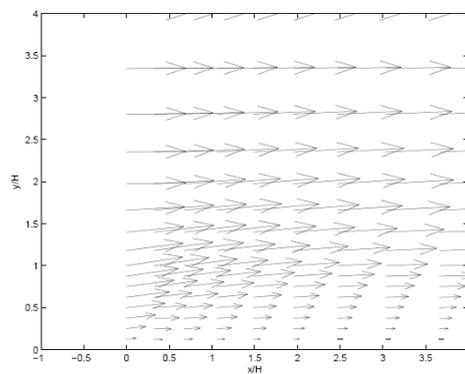


Figure 7. Details of the velocity field on the forest edge.

The second set of results was obtained for a test case concerning the development of boundary layer flow after a forest edge, as shown in Fig. 5. The canopy has a height  $H$ , and the computation domain is contained in a box with

18Hx3H. The velocity field is developed from the beginning of the canopy layer ( $x = 0$ ) just at a condition with zero vertical velocity and an established vertical profile of the wind velocity. The domain is discretized with 2200 triangular finite elements and the measured canopy structures is used for the simulations ( $a(z)$ ). The simulation is performed for the uniform plantation of Sitka spruce to verify the development of the air flow within the canopy, near to the forest edge. Experimental results are available for this situation.

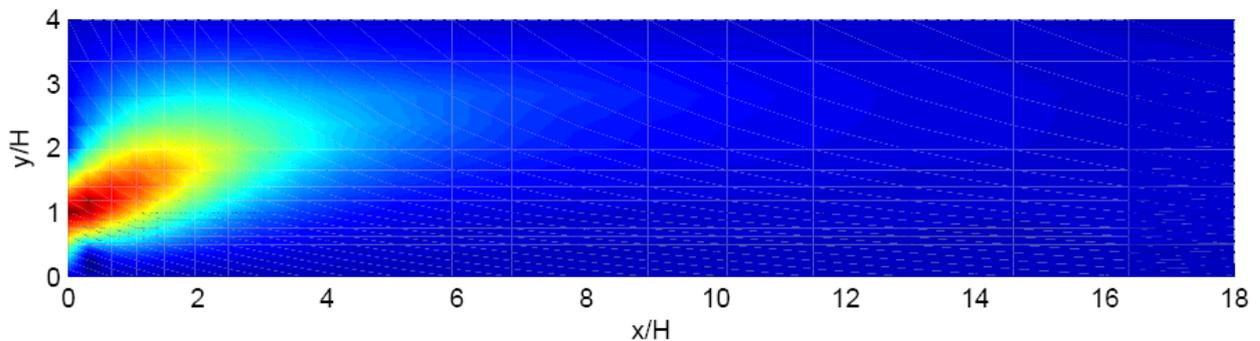


Figure 8. Vertical velocity levels

On Figures 6-8 the numerical results for the developing flow in the forest edge are visualized. The velocity vectors characterized the same qualitative streamlines behavior of the experimental observations of Irvine et al. (1997) and Morse et al. (2002). Near the inlet forest edge, the vertical velocity component of the velocity has a great positive value, as a consequence of the deceleration of the air flow due to the drag in the vegetation elements. At this region the velocity has an angle of some degrees, directing the flow to the top of the canopy, as shown in the pictures. It can be observed that the results are overestimated within the canopy, comparing it to the experimental observations. Two considerations have to be taken into account: First, the exact leaf area density distribution is only estimated in this experimental case. The data for the function  $a(x)$  is not available for this experiments - only the integral value (LAI) had been effectively measured. The authors proposed an estimative as a conical distribution, which can be explain the difference between the results. The second consideration is based in the modeling used for this paper. The simply first order model considers a constant length scale within the canopy, and the computation of the turbulence viscosity is performed and calibrated only for the developed boundary layer flows. Advanced models have to be used to enhance the accuracy of the estimates for the air flow, for inhomogeneous distributions of vegetative layer, in particular to describe the flow in near ground regions.

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## 6. References

- Codina, R., Vazquez, M. and Zienkiewicz, O.C., 1998, A general algorithm for compressible and incompressible flow - part III. The semi-implicit form, *International Journal for Num. Methods in Fluids* Vol. 27, pp. 13–32.
- Codina, R., 2001, Pressure stability in fractional step finite element methods for incompressible flows, *J. of Comp. Physics*, Vol. 170, pp. 112–140.
- Codina, R. and Zienkiewicz, O. C., 2002, CBS versus GLS stabilization of the incompressible Navier-Stokes equations and the role of the time step stabilization parameter, *Comm. in Num. Methods in Engineering*, Vol. 18, pp. 99–112.
- Chorin, A.J., 1968, Numerical solution of the Navier-Stokes equations, *Mathematics of Computation*, Vol. 22, pp. 745–762.
- Finnigan, J.J., 2000, Turbulence in plant canopies, *Annual Review of Fluid Mechanics*, Vol. 32, pp. 519–571.
- Katul, G.G., Mahrt, H., Poggi, D. and Sanz, C. 2004, One- and Two-Equation Models for Canopy Turbulence, *Boundary-Layer Meteorology*, Vol. 113, pp. 81-109.
- Kobayashi, Pereira, J. and Siqueira, M., 1994, Numerical Study of the Turbulent Flow over and in a Model Forest on a 2D Hill, *J. Wind Eng. Ind. Aerodyn.* Vol. 53, pp. 357-374.
- Irvine, M.R., Gardiner, B.A. and Hill, M.K., 1997, The Evolution of Turbulence Across a Forest Edge, *Boundary-Layer Meteorology*, Vol. 84, pp. 467-496.

- Lee, X., 2000, Air motion within and above forest vegetation in non-ideal conditions, *Forest Ecology and Management* Vol. 135, pp. 2–18.
- Morse, A.P., Gardiner, B.A. and Marshall, B.J., 2002, Mechanisms Controlling Turbulence Development Across a Forest Edge, *Boundary-Layer Meteorology*, Vol. 103, pp. 227-251.
- Pinard, J.P. and Wilson, J.D., 2001, First- and Second-Order Closure Models for Wind in a Plant Canopy, *J. Appl. Meteorology*, Vol. 40, pp. 1762–1768.
- Poggi, D., Katul, G.G. and Albertson, J.D., 2004, Momentum Transfer and Turbulent Kinetic Energy Budgets within a Dense Canopy, *Boundary-Layer Meteorology*, Vol. 111, pp. 589–614.
- Raupach, M.R and Shaw, R.H., 1982, Averaging Procedures for Flow Within Vegetation Canopies, *Boundary-Layer Meteorology* Vol.2(1), pp. 79–90.
- Ross, A.N., Arnold, S., Vosper, S.B., Mobbs, S.D. Dixon, N. and Robins, A.G., 2004, A Comparison of Wind-Tunnel Experiments and Numerical Simulations of Neutral and Stratified Flow Over A Hill, *Boundary-Layer Meteorology* Vol. 113, pp. 427-459.
- Ross, A.N and Vosper, S.B., 2004, Turbulent flow over a forested hill. Part 1: Numerical modelling and comparison with experimental data, *Q. J. R. Meteorol. Soc.* Vol. 113, pp. 427-459.
- Temam, R., 1969, Sur l'approximation de la solution des équations de Navier-Stokes par la Méthode de pas fractionnaires, *Archives for Rational Mechanics and Analysis*, Vol. 32, pp. 135– 153.
- Wilson, J.D. and Flesch, T.K., 1999, Wind and remnant tree sway in forest cutblocks. III. a wind flow model to diagnose spatial variations, *Agricultural and Forest Meteorology* Vol. 93, pp. 259–282.
- Zienkiewicz, O.C and Taylor, R.L., 2000, *The Finite Element Method - Fluid Mechanics*, Butterworth-Heinemann Pub., Vol. 3.

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